

Null electromagnetic fields destroy black holes

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We consider two test, null electromagnetic fields aligned with the two repeated principal null directions of the type D background, and a test, non-null field special in the sense that the principal null directions of the electromagnetic field lie along the repeated principal null directions of the space-time. We show that the interaction of the null field along l^a with Kerr black holes leads to violation of two fundamental laws/conjectures of black hole physics: a generic violation of cosmic censorship conjecture, and violation of the area theorem. These results are totally unexpected considering the fact that the energy-momentum tensor obeys the weak energy condition. We also show that the special non-null field does not lead to any perturbation of black hole parameters of mass and angular momentum, and the null field directed along n^a does not challenge cosmic censorship or the area theorem.

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I. INTRODUCTION

According to singularity theorems of Hawking and Penrose, the formation of singularities is inevitable as a result of gravitational collapse in classical general relativity, given some very reasonable assumptions [1]. In the model developed by Penrose and Hawking the trapped surface that arises in the spherically symmetric gravitational collapse of a body, is contained in the black hole region of the space-time, so the singularity is surrounded by an event horizon. This singularity can be considered harmless for distant observers, as opposed to a naked one which intersects a Cauchy surface rendering the initial conditions undefined, thus disabling asymptotic predictability. Penrose proposed the Cosmic Censorship Conjecture (CCC) [2], in order to avoid these pathologies and preserve the deterministic nature of general relativity. The weak form (WCCC) asserts that the singularities that arise in gravitational collapse are always hidden behind event horizons. In this respect, a distant observer does not encounter singularities or any effects propagating out of them, and the consistency of the theory of general relativity is reassured at least in the space-time excluding the black hole region bounded by the event horizon.

It has not been possible to establish a concrete proof of CCC. For that reason Wald constructed an alternative thought procedure to test the stability of event horizons [3]. Consider a Kerr-Newman black hole defined by three parameters (Mass M , charge Q , and angular momentum per unit mass a), which should satisfy

$$M^2 \geq Q^2 + a^2. \quad (1)$$

so that an event horizon exists. Then the black hole is allowed to absorb test particles or fields incident from

infinity. By no hair theorem, the space-time settles to another stationary condition with new parameters M, Q, a at the end of the interaction. If the final configuration of parameters satisfies (1), the interaction with test fields or particles has not destroyed the horizon, so the CCC remains valid. If the final configuration does not satisfy (1), the black hole has turned into a naked singularity and CCC is violated. In the first example of these thought experiments Wald showed that particles with enough charge or angular momentum to destroy the horizon either miss or are repelled by the black hole. Many authors followed Wald to construct similar thought experiments to test the validity of CCC in the interaction of black holes with test particles or fields. [4–26]

Recently we have constructed a thought experiment [27] in which a free test electromagnetic field interacts with an extremal Kerr black hole, and showed that CCC remains valid in this case. In this work we check the validity of CCC for the special electromagnetic fields interacting with Kerr black holes. We consider two test, null electromagnetic fields aligned with the two repeated principal null directions of the type D background, and a test, non-null field, special in the sense that the principal null directions of the electromagnetic field lie along the repeated principal null directions of the space-time. In Newman Penrose (NP) two spinor formalism [28] these fields correspond to special solutions of Maxwell equations with a single non-vanishing NP Maxwell scalar. In sections II and III we review the existence and behaviour of these fields in a type D vacuum background. In section IV we test the validity of CCC in the interaction of black holes with special electromagnetic fields.

A. Electromagnetic and gravitational perturbations in Newman Penrose formalism

Newman Penrose (NP) two spinor formalism [28] has proved very useful in studying the perturbations and asymptotic structure of space-times. This formalism is

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based on a spin basis (o, ι) endowed with a symplectic structure $\epsilon_{AB} = -\epsilon_{BA}$. In NP formalism the source free Maxwell equations have the form:

$$\nabla^{AA'} \phi_{AB} = 0 \quad (2)$$

There are four complex equations here by $A', B = 0, 1$, corresponding to eight real Maxwell equations. The symmetric valence 2 spinor ϕ_{AB} generates 3 complex scalars via

$$\phi_0 = \phi_{AB} o^A o^B \quad \phi_1 = \phi_{AB} o^A \iota^B \quad \phi_2 = \phi_{AB} \iota^A \iota^B \quad (3)$$

The NP description of electromagnetism is given in terms of these scalars (3). Also note that

$$\phi_{AB} = \phi_2 o_A o_B - 2\phi_1 o_{(A} \iota_{B)} + \phi_0 \iota_A \iota_B \quad (4)$$

The explicit form of Maxwell's equations in terms of NP scalars can be derived.

$$\begin{aligned} (D - 2\rho)\phi_1 - (\bar{\delta} + \pi - 2\alpha)\phi_0 + \kappa\phi_2 &= 0 \\ (D - \rho + 2\epsilon)\phi_2 - (\bar{\delta} + 2\pi)\phi_1 + \lambda\phi_0 &= 0 \\ (\delta - 2\tau)\phi_1 - (\Delta + \mu - 2\gamma)\phi_0 - \sigma\phi_2 &= 0 \\ (\delta - \tau + 2\beta)\phi_2 - (\Delta + 2\mu)\phi_1 - \nu\phi_0 &= 0 \end{aligned} \quad (5)$$

Any totally symmetric spinor can be decomposed in terms of univalent spinors (see e.g. [29, 30]). Hence we may decompose the spinor equivalent of Maxwell tensor.

$$\phi_{AB} = \alpha_{(A} \beta_{B)} \quad (6)$$

α and β are called the principal spinors of ϕ_{AB} . If α and β are proportional then α is called a repeated principal spinor of ϕ , and ϕ is said to be algebraically special or null, or of type N . The corresponding real null vector $\alpha_a = \alpha_A \bar{\alpha}_{A'}$ is called a repeated principal null direction. If α and β are not proportional then ϕ is said to be algebraically general or type I , or non-null.

To formulate gravity let us define the spinor equivalent of the Weyl tensor C_{abcd} .

$$C_{abcd} + iC_{abcd}^* = 2\Psi_{ABCD}\epsilon_{A'B'}\epsilon_{C'D'} \quad (7)$$

Ψ_{ABCD} is totally symmetric and satisfies the spinor analogue of Bianchi identities in vacuum

$$\nabla^{DD'} \Psi_{ABCD} = 0 \quad (8)$$

The explicit form of Bianchi identities are derived in [28]. (Also see [29, 30]) Since Ψ_{ABCD} is totally symmetric there exists univalent spinors $\alpha_A, \beta_B, \gamma_C, \delta_D$ such that

$$\Psi_{ABCD} = \psi \alpha_{(A} \beta_B \gamma_C \delta_{D)} \quad (9)$$

$\alpha_A, \beta_B, \gamma_C, \delta_D$ are called the principal spinors of Ψ_{ABCD} . The corresponding real null vectors determine the principal null directions of Ψ_{ABCD} . The classification of space-times according to the principal null directions of the Weyl tensor is known as Petrov classification [31]. As in the case of electromagnetism if none of the principal null directions coincide the space-time is algebraically general of type I , and if all four principal null directions coincide the space-time is of type N . If there are two pairs of repeated principal null directions the space-time is of type D .

II. EXISTENCE OF SPECIAL FIELDS IN TYPE D SPACE-TIMES

Algebraically special (null) electromagnetic fields are known to exist in space-times that admit a shear free, geodesic null congruence [32]. Existence of the fields refers to the fact that the integrability conditions are satisfied by Maxwell equations (also see e.g. [30]). Let us consider a null electromagnetic field ϕ_{AB} and choose a spin basis (o, ι) such that $\phi_{AB} = \phi o_A o_B$. From (4) it follows that the only non-vanishing Maxwell scalar is $\phi_2 = \phi$. With our choice of spin basis the null congruence has tangent vector l^a , and the fact that it is geodesic and shear free implies $\kappa = \sigma = 0$. We also choose $\epsilon = 0$ by affine parametrization of the geodesic. Maxwell equations (5) reduce to

$$D\phi = \rho\phi, \quad \delta\phi = (\tau - 2\beta)\phi \quad (10)$$

A solution for (10) does not necessarily exist. One can show that a solution exists if the background space-time satisfies ($\kappa = \sigma = 0$); i.e. the space-time admits a geodesic, shear-free null congruence. (see e.g. [30])

There exists another null field on a type D background, which lies along n^a . For this field $\phi_{AB} = \phi \iota_A \iota_B$, and the only non-vanishing Maxwell scalar is $\phi_0 = \phi$. The Maxwell equations for this field reduce to

$$\bar{\delta}\phi = (2\alpha - \pi)\phi \quad \Delta\phi = (2\gamma - \mu)\phi \quad (11)$$

One can also show that the integrability conditions for (11) are identically satisfied if the space-time that admits a geodesic and shear free null congruence with the tangent vector n^a ($\lambda = \nu = 0$).

The two principal null directions of Petrov type D space-times determine two geodesic and shear free null congruences with tangent vectors l^a and n^a . For that reason two null electromagnetic fields with principal null directions l^a and n^a exist in type D space-times, such that the only non-vanishing Maxwell scalars are ϕ_2 and ϕ_0 , respectively. This should not be confused with the case that a single null field is expressed in terms ϕ_2 or ϕ_0 by transformation of spin basis. The two null fields in type D space-times exist independently and simultaneously.

In addition to null fields, we consider an algebraically general (non-null) test electromagnetic field in type D vacuum space-times, which is special in the sense that the principal null directions of the electromagnetic field lie along the repeated principal null directions of the space-time. It is known that electrovacuum solutions for type D space-times exist such that the two repeated principal null congruences of the Weyl tensor are aligned with the two principal null congruences of the non-null electromagnetic field [33]. (Also see [34] and references therein) But a reference for the proof that type D vacuum space-times admit such special non-null test fields is not known to this author, so we prove it here.

Theorem II.1. *Type D vacuum space-times admit a special, test, non-null electromagnetic field, such that the two principal null directions of the electromagnetic field lie along the repeated principal null directions of the space-time.*

Proof. Naturally we choose a spinor basis (o, ι) for a type D space-time such that the two principal null directions correspond to $l^A = o^A o^{A'}$ and $n^A = \iota^A \iota^{A'}$. Consider an algebraically general test electromagnetic field

$$\phi_{AB} = \phi o_{(A} \iota_{B)} \quad (12)$$

which is special in the sense that its principal null directions are parallel to those of the background space-time. (A test field is one that has a negligible effect on the background geometry.) From (3) and (4) we see that $\phi_0 = \phi_2 = 0$. The only non-vanishing Maxwell scalar is ϕ_1 , which equals $-\phi/2$ according to our definition (12). Then, Maxwell equations have the form:

$$\begin{aligned} D\phi_1 &= 2\rho\phi_1 \\ \Delta\phi_1 &= -2\mu\phi_1 \\ \delta\phi_1 &= 2\tau\phi_1 \\ \bar{\delta}\phi_1 &= -2\pi\phi_1 \end{aligned} \quad (13)$$

We have to prove that a solution for the system (13), i.e. a solution of Maxwell's equation such that the only non-vanishing Maxwell scalar is ϕ_1 , exists in a type D background. If a space-time is of type D the only non-vanishing scalar of the Weyl tensor is Ψ_2 . In this case the Bianchi identities in vacuum reduce to

$$\begin{aligned} D\Psi_2 &= 3\rho\Psi_2 \\ \Delta\Psi_2 &= -3\mu\Psi_2 \\ \delta\Psi_2 &= 3\tau\Psi_2 \\ \bar{\delta}\Psi_2 &= -3\pi\Psi_2 \end{aligned} \quad (14)$$

The integrability conditions for the systems (13) and (14) are identical. In other words the integrability conditions for the existence of a special test electromagnetic field in the form (12), in a type D vacuum background, are identical with the conditions for the existence of the type D background itself. Thus, there exists a special non-null test electromagnetic field in every type D vacuum background. \square

The integrability conditions for the existence of type D vacuum space-times were derived by Kinnersley [35]. The same conditions assure the existence of a solution for the system (13), on a type D vacuum background.

III. BEHAVIOUR OF SPECIAL FIELDS IN KERR SPACE-TIME

In section (II) we proved the existence of a special electromagnetic field in type D space-times, such that the only non-vanishing Maxwell scalar is ϕ_1 . We are particularly interested in its asymptotic behaviour in Kerr

space-time. From NP field equations for type D space-times, we have $D\rho = \rho^2$. This leads to a solution for Bianchi identities $\Psi_2 = \rho^3 \Psi^0$, where Ψ^0 is a constant [35]. In particular $\Psi_2 = M\rho^3$ for a Kerr black hole (see e.g. [36]). Similarly the solution for Maxwell equations with $\phi_0 = \phi_2 = 0$ is given by

$$\phi_1 = \rho^2 C_1 \quad (15)$$

where C_1 is independent of r . In Kerr space-time $\rho = -(r - ia \cos \theta)^{-1}$ so the special field behaves as $1/r^2$ everywhere. In fact, the general expression for ρ in a type D vacuum space-time is $\rho = -(r + i\rho^0)^{-1}$, where ρ^0 is independent of r [35], so $1/r^2$ behaviour applies to every type D vacuum space-time.

We are particularly interested in the flux of energy and angular momentum at large distances appropriate for wave propagation, both for the outgoing and ingoing waves. The formal expression for total energy flux per unit solid angle is (see e.g. [36])

$$\frac{dE}{dt d\Omega} = \lim_{r \rightarrow \infty} r^2 T^1_0 \quad (16)$$

T^1_0 can be substituted by T^1_3 to give angular momentum flux. For the limits to exist, T^1_0 and T^1_3 components of the energy momentum tensor must have $1/r^2$ behaviour at large distances. The energy momentum tensor for electromagnetic fields in terms of Maxwell's scalars, is given by

$$\begin{aligned} 4\pi T_{\mu\nu} &= \{\phi_0 \phi_0^* n_\mu n_\nu + 2\phi_1 \phi_1^* [l_{(\mu} n_{\nu)} + m_{(\mu} m_{\nu)}^*] + \phi_2 \phi_2^* l_\mu l_\nu \\ &\quad - 4\phi_1 \phi_0^* n_{(\mu} m_{\nu)} - 4\phi_2 \phi_1^* l_{(\mu} m_{\nu)} + 2\phi_2 \phi_0^* m_\mu m_\nu\} \\ &\quad + \text{c.c.} \end{aligned} \quad (17)$$

where c.c. denotes complex conjugate. For the special non-null field introduced in section II the energy momentum tensor reduces to

$$4\pi T_{\mu\nu} = 2\phi_1 \phi_1^* [l_{(\mu} n_{\nu)} + m_{(\mu} m_{\nu)}^*] + \text{c.c.} \quad (18)$$

Considering $1/r^2$ behaviour of the special field, T^1_0 and T^1_3 behave as $1/r^4$, therefore the flux of energy and angular momentum due to the special electromagnetic field vanishes in the limit $r \rightarrow \infty$. Also consider the NP tetrad for Kerr metric

$$\begin{aligned} l^\mu &= [(r^2 + a^2)/\Delta, 1, 0, a/\Delta] \\ n^\mu &= [(r^2 + a^2), -\Delta, 0, a]/(2\Sigma) \\ m^\mu &= [ia \sin \theta, 0, 1, i/\sin \theta]/[\sqrt{2}(r + ia \cos \theta)] \end{aligned} \quad (19)$$

where $\Sigma = r^2 + a^2 \cos^2 \theta$ and $\Delta = r^2 - 2Mr + a^2$. Using this tetrad we see that the expression $[l_{(1} n_{0)} + m_{(1} m_{0)}^*] = [l_{(1} n_{3)} + m_{(1} m_{3)}^*] = 0$. This implies that T^1_0 and T^1_3 vanish everywhere if ϕ_1 is the only non-vanishing Maxwell scalar for an electromagnetic field in Kerr space-time, regardless of the solution for the field. Thus, the contribution of the special electromagnetic field introduced in section (II) to the radial energy and angular momentum fluxes vanishes everywhere, in Kerr background.

Let us evaluate the behaviour of null fields in Kerr background. The nonvanishing spin coefficients are

$$\begin{aligned}\rho &= -1/(r - ia \cos \theta), \quad \beta = -\rho^* \cot \theta / 2\sqrt{2} \\ \tau &= ia\rho\rho^* \sin \theta / \sqrt{2}, \quad \gamma = \mu + \rho\rho^*(r - M)/2 \\ \pi &= ia\rho^2 \sin \theta / \sqrt{2}, \quad \mu = \rho^2\rho^*\Delta/2, \quad \alpha = \pi - \beta^*(20)\end{aligned}$$

For the null field which lies along l^a , (using $D\rho = \rho^2$) the solution for $(D - \rho)\phi_2 = 0$ is

$$\phi_2 = \rho C_2 \quad (21)$$

so ϕ_2 behaves as $1/r$. T^1_0 and T^1_3 behave as $1/r^2$, as required by (16) so that the contribution of the null field to energy and angular momentum fluxes can be calculated.

For the null field along n^a , we have $\Delta\phi_0 = (2\gamma - \mu)\phi_0$. As $r \rightarrow \infty$, $\rho \sim \rho^*$, $\mu \sim \mu^* \sim \rho/2$, and $\gamma \sim \gamma^* \sim 0$. So the Maxwell equation is reduced to

$$\lim_{r \rightarrow \infty} \Delta\phi_0 \sim (-1/2)\rho\phi_0 \quad (22)$$

Consider the following NP field equation in a vacuum type D space-time

$$\Delta\rho - \bar{\delta}\tau = -\rho\bar{\mu} + (\bar{\beta} - \alpha - \bar{\tau})\tau + (\gamma + \bar{\gamma})\rho - \Psi_2 \quad (23)$$

As $r \rightarrow \infty$, $\tau \sim \rho^2 \sim 0$, and $\Psi_2 \sim \rho^3 \sim 0$. The NP field equation (23) is reduced to

$$\lim_{r \rightarrow \infty} \Delta\rho \sim (-1/2)\rho^2 \quad (24)$$

Using (22) and (24) we see that

$$\lim_{r \rightarrow \infty} \phi_0 \sim \rho C_0 \quad (25)$$

ϕ_0 also behaves as $1/r$ and therefore satisfies the limit condition (16) so that its contribution to fluxes of energy and angular momentum can be calculated.

IV. SPECIAL ELECTROMAGNETIC FIELDS AND COSMIC CENSORSHIP

A Kerr black hole is uniquely parametrised by its mass M and angular momentum a . The changes in the black hole parameters can be expressed as fluxes into the black hole. Let K be a Killing vector. The Killing equation, $\nabla_{(a}K_{b)} = 0$, combined with the local conservation of energy-momentum $\nabla_c T^{ac} = 0$ in general relativity, leads to the current conservation equation $\nabla_a(T^{ac}K_c) = 0$. For $K = \partial/\partial t$ we have

$$\left(\frac{dM}{dt}\right)_{\text{b.h.}} = - \int_{S_\infty} \sqrt{-g} T^1_0 d\theta d\phi \quad (26)$$

and for $K = \partial/\partial \phi$

$$\left(\frac{dL}{dt}\right)_{\text{b.h.}} = \int_{S_\infty} \sqrt{-g} T^1_3 d\theta d\phi \quad (27)$$

where the label b.h. refers to the black hole and S_∞ is the spherical surface as $r \rightarrow \infty$. Next we follow [6] to define an indicator for CCC

$$\mathcal{C} = M^2 - a^2 \quad (28)$$

Then, using $a = L/M$

$$\delta\mathcal{C} = \int \frac{d\mathcal{C}}{dt} dt = \int \frac{2}{M} \left\{ (M^2 + a^2) \frac{dM}{dt} - a \frac{dL}{dt} \right\} dt \quad (29)$$

implying

$$\frac{d\mathcal{C}}{dt} = \frac{2}{M} \int_S \sqrt{-g} [(M^2 + a^2)(-T^1_0) - aT^1_3] d\theta d\phi \quad (30)$$

An extremal black hole would saturate the main criterion (1). In that case $\delta\mathcal{C}$ should always remain positive so that the event horizon is preserved. If the initial state is an extremal black hole and $\delta\mathcal{C}$ turns out to be negative, the final state describes a naked singularity.

The validity of CCC in the case of the special non-null field is trivial. For this field we have $T^1_0 = T^1_3 = 0 \Rightarrow \delta\mathcal{C} = 0$. The non-null field does not lead to any perturbation of black hole parameters of mass and angular momentum, hence does not challenge CCC whether we start with an extremal or a nearly extremal black hole.

Let us now consider a Kerr black hole interacting with the null field that lies along n^a . For this field the only non-vanishing Maxwell scalar is ϕ_0 and the energy-momentum tensor (17) reduces to

$$T_{\mu\nu} = \frac{1}{2\pi} |\phi_0|^2 n_\mu n_\nu \quad (31)$$

By direct substitution from the NP tetrad (19)

$$2\pi T^1_0 = -|\phi_0|^2 \frac{\Delta^2}{4\Sigma^2} \quad 2\pi T^1_3 = |\phi_0|^2 a \sin^2 \theta \frac{\Delta^2}{4\Sigma^2} \quad (32)$$

Note that as $r \rightarrow \infty$, $(\Delta^2/4\Sigma^2) \sim 1$. Then

$$\delta\mathcal{C} = \frac{2}{M} \int_{S_\infty} \frac{\sqrt{-g}}{2\pi} \frac{|\phi_0|^2}{4} (M^2 + a^2 \cos^2 \theta) d\theta d\phi dt \quad (33)$$

The expression (33) is positive definite. The result is independent from the specific form of ϕ_0 provided that the limit condition (16) is satisfied. Thus, whether we start with an extremal or a nearly extremal Kerr black hole, CCC always remains valid in the interaction of the black hole with the null field along n^a .

Now let us consider the interaction of the black hole with the null field along l^a . For this field the only non-vanishing Maxwell scalar is ϕ_2 , and the energy momentum tensor (17) reduces to

$$T_{\mu\nu} = \frac{1}{2\pi} |\phi_2|^2 l_\mu l_\nu \quad (34)$$

Again we make a direct substitution from the NP tetrad (19)

$$2\pi T^1_0 = |\phi_2|^2 \quad 2\pi T^1_3 = -(a \sin^2 \theta) |\phi_0|^2 \quad (35)$$

CCC indicator takes the form

$$\delta\mathcal{C} = \frac{2}{M} \int_{S_\infty} \frac{\sqrt{-g}}{2\pi} (-|\phi_2|^2) (M^2 + a^2 \cos^2 \theta) d\theta d\phi dt \quad (36)$$

The expression (36) is negative definite, independent from the specific form of ϕ_2 . The interaction with the null field, decreases the values of both mass and angular momentum parameters of the black hole; but mass loss exceeds angular momentum loss. (substituting (35) in (26) and (27), we obtain negative values for both dM/dt and dL/dt) As a result the value of $M^2 - a^2$ decreases. Thus, if we start with an extremal black hole ($M^2 - a^2 = 0$), the null field along l^a turns it into a naked singularity, and CCC is violated. So far, the only context where a thought experiment can result in the destruction of an extremal black hole without fine-tuning was due to neutrino fields [21]. Also, there exists claims of destruction of extremal black holes with finely tuned particles/fields [19], but these are expected to be challenged by backreaction or self-force effects.

Since $\delta\mathcal{C}$ is always negative, if we start with a nearly extremal black hole the interaction with the null field along l^a drives it to extremality and beyond. The fact that $\delta\mathcal{C}$ is always negative also implies that the initial data giving rise to violation is not confined to a set of measure zero. In this sense the violation of CCC by null fields along l^a is generic. Such a generic violation occurs in the case of neutrino fields where the black hole is overspun if the incoming field is in the frequency range $0 < \omega < m\Omega$. However, the most generic violation of CCC among all the thought experiments involving particles and fields turns out to be the case of null electromagnetic fields, since no initial conditions has to be imposed on the thought experiment to ensure that $\delta\mathcal{C}$ is negative.

V. NULL FIELDS AND BLACK HOLE MECHANICS

The violation of CCC by null electromagnetic fields and neutrino fields have common aspects such as being generic and applying to extremal black holes. However, the energy momentum tensor for neutrino fields (or Dirac fields in general) fundamentally differs from that of bosonic fields in the sense that it does not satisfy the weak energy condition. For that reason the area theorem is not expected to hold for Dirac fields. Since, the change in the area of the black hole is given by $dA = (8\pi/\kappa)(dM - \Omega dJ)$, the absorption of the modes $0 < \omega < m\Omega$ decreases black hole's area, violating the second law of mechanics. The absorption of the same modes leads to violation of CCC [21, 24]. This result can also be considered as a consequence of the fact that the energy momentum tensor does not satisfy the null energy condition.

On the other hand the energy-momentum tensor for bosonic fields satisfies the null energy condition, and the area theorem is expected to hold [37]. This will forbid

the formation of a naked singularity, since the area of the event horizon cannot decrease. Recently it was shown that test fields that satisfy the null energy condition, cannot destroy extremal black holes via a derivation that relies on black hole thermodynamics [26]. In that case it may not be clear how to explain the generic violation of CCC by null electromagnetic fields. Let us check if the area theorem actually holds for the null field along l^a . The area of the horizon is given by

$$A = \int_{r=r_+} \sqrt{g_{\theta\theta}g_{\phi\phi}} d\theta d\phi = 8\pi M \left(M + \sqrt{M^2 - a^2} \right) \quad (37)$$

Using (26) and (35)

$$(dM)_{b.h} = - \int_{S_\infty} \sqrt{-g} |\phi_2|^2 d\theta d\phi dt < 0 \quad (38)$$

Note that $d(M^2 - a^2)$ is $\delta\mathcal{C}$ in our notation and we have shown that $\delta\mathcal{C} < 0$ in the previous section (36). Having both $dM < 0$ and $d(M^2 - a^2) < 0$ in eq. (37), directly leads to $dA < 0$. Thus, the area theorem does not hold in the interaction of the null field that lies along l^a with the black hole, although the energy-momentum tensor for the field satisfies the null energy condition. Null fields not only violate CCC but also the second law of black hole mechanics. For that reason the derivation in [26] does not apply to null fields.

For the sake of completeness, we should also evaluate the validity of the area theorem in the interaction of the black hole with the null field along n^a . Substituting (32) in (26) gives $dM > 0$, and (33) implies $d(M^2 - a^2) > 0$. Thus, $dA > 0$ and the area theorem holds.

VI. SUMMARY AND CONCLUSIONS

In this paper, we have shown that the interaction of a Kerr black hole with the null electromagnetic field such that the only non-vanishing Maxwell scalar is ϕ_2 , leads to violation of two fundamental laws/conjectures of black hole physics. First, we have proved that a generic violation of cosmic censorship conjecture occurs. If the initial state is an extremal black hole the horizon is destroyed, if it is a sub-extremal black hole the interaction drives it to extremality and beyond. Such a generic violation which applies to extremal black holes was shown to occur in the interaction with neutrino fields [21, 24]. However, this violation can be expected since the energy-momentum tensor for the neutrino fields does not satisfy the weak energy condition. On the contrary bosonic fields satisfy the weak energy condition, and the area theorem is expected hold for these fields which would forbid such a generic destruction of the event horizon. In this work we have shown that the area theorem does not hold either for the null electromagnetic field that lies along the principal null direction l^a . These violations are evidently perplexing considering the fact that the energy-momentum

tensor for the field satisfies the weak/null energy condition.

We have also considered the null field that lies along the principal null direction n^a , and a special non-null field such that the principal null directions of the field lie along the repeated principal null directions of the space-time. We proved the existence of this field on type vacuum background. We showed that the contribution of the non-null field to mass and angular momentum parameters of a Kerr black hole identically vanishes, and the null field

along n^a does not challenge cosmic censorship conjecture or the area theorem.

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